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Estimating Yield Curve The Svensson Extended Model Using L-BFGS-B Method Approach

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Abstract. Yield curve is curves that describe the magnitude of the yield against maturity. To describe this curve, we use the Svensson model. One extension of this model is Rezende-Ferreira. Expansion undertaken by Rezende-Ferreira has weaknesses that there are several parameters have the same value. These values form Nelson-Siegel model. In this paper, we propose expansion of Svensson model. These models are non-linear model, so it is more difficult to estimate. To overcome this problem, we propose Nonlinear Least Square by L-BFGS-B method approach.

Keywords: Yield curve, Svensson extended, and L-BFGS-B.
PACS: 00-General

INTRODUCTION

Investing in bonds, we need information about the yield that would be obtained and the maturing of the bond. This component is the main component in bound investing. These two components is also foundation for researchers to describe the yield curve.

Yield curve can be described by various model, one of which is Nelson-Siegel class model. This model is experiencing rapid growth which is widely used by various states as reference in the view of the yield curve. The models included in Nelson-Siegel class model that is Nelson-Siegel (NS) model, Svensson (SV) model, Rezende-Ferreira (RF) model, and Svensson extended (SVE) model. SVE model is a model that extended of SV model by combining the first hump and hump two in one factor, so this model consists of flat, slope, hump combined, and the second hump.

The extension of this model is based on the RF by adding second slope into SV model resulting model will form NS model until the addition of the slope does not give accuracy from previous models (SV model). RF model is a model consisting of flat curve, first slope, second slope, first hump, and second hump (Rezende and Ferreira (23)). This model is extension of the SV model, SV model consist of flat curve, slope, first hump and second hump (Svensson (26)).

These models are based on earlier model which consists of flat, slope, and hump proposed by Nelson and Siegel of the University of Washington in 1987 (Nelson and Siegel (21)). In this paper, we just depictions of the yield curve and do not forecasting of yield curve. Papers that discuss the Nelson-Siegel class model, namely: Svensson (26); Boldier and Streliski (2); Mansi and Philips (20); Brousseau (3); Jankowitsch and Pictler (16); Diebold, et al (10); Diebold et al (6); Diebold, et al (8); Diebold, et al (9); Krippner (18); Ejsing, et al (12); Bauer (1); Christensen, et al (4); Christensen (5); Kripner (19); Ferst and Hayden (13); Gilli, et al (14); Rezende and Ferreira (23); and Rosadi (24).

Nelson-Siegel class model has linear and nonlinear parameters. In these conditions of these models has many local minima so that the estimate can't use the usual estimate. Previous studies have been widwly discussed estimation of the Nelson-Siegel class model, including: Boldier and Streliski (2) discusses this model in various maturity by estimate discuss the maximum likelihood estimation (MLE), Maria, et al (28) compared the Nelson-Siegel Model and Vasicek models, the estimate used is MLE. Gilli, et al (14) estimate the Svensson models with Least Square algorithm in two forms the optimization: first optimization of curvature has many local optima, second particular emphasis on a range of parameters, the conditions are bad models provide estimates of the parameters are not stable against the data. Rezende and Ferreira (23) compare the forecasting of the four development Nelson-Siegel model class, namely: NS, Bliss, SV and the five-factor model in improving the model of perfection, which is to use innovation to quantile Autoregression (QAR). Rosadi (24) doing SV model estimation by PQRT using a R computational.

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This model has many local minima, in this condition to resolve the issue. We propose the OLS estimation by L-BFGS-B method optimization approach. This optimization method is an extension of the limited memory BFGS method (LM-BFGS or L-BFGS) that uses the simple boundaries of the model Zhu, et al (27)). This method can solve the problem that has many local minima of a model. BFGS is an combination of development pioneered by Broyden, Fletcher, Goldfarb, and Shanno. In this paper, we study mathematically not provided, an explanation of this method can be seen in Griva, et al (15), Kelly (17), Sun and Yuan (25), and Rao (29).

MODEL

In this section will discuss the Nelson-Siegel class model and its development. The models are used to describe the yield curve. This model is a parametric model that has several factors. Nelson-Siegel class model discussed were:

NS model was first developed by Charles Nelson and Andrew Siegel of the University of Washington in 1987. This modeling is based on various forms of maturity that describe the yield curve, such as: flat shape, hump shape, and S shape (Nelson (21)). This model is formulated as:

$$y(\lambda, \dot{\theta}) = \beta_0 + \beta_1\, exp\left(-\frac{\lambda}{\tau}\right) + \beta_2 \frac{\lambda}{\tau} \exp\left(-\frac{\lambda}{\tau}\right), \tag{1}$$

Were y is yield, λ is maturity, $\boldsymbol{\beta}$ is linear parameters vector, i.e. $\boldsymbol{\beta} = (\beta_0,\ \beta_1,\ \beta_2)'$, τ is nonlinear parameter of maturity, and $\dot{\theta} = (\boldsymbol{\beta}, \tau)'$. β_0 is function early maturing, will be worth anyway if time to maturity approaches zero, β_1 determine the beginning of the curve (short) in a variety of irregularities, this curve will be negatively skewed if the positive parameters, yield function will converge to $\beta_0 + \beta_1$ if λ approaching infinity, β_2 determine the magnitude and direction of the hump, and τ determine the position of the hump or U shape curve. [26] to develop a model NS by adding a second hump in the model so that it becomes a model four-factor models, better known by SV. This model is written as

$$y\left(\lambda, \ddot{\theta}\right) = \beta_0 + \beta_1\, exp\left(-\frac{\lambda}{\tau_1}\right) + \beta_2 \left[\frac{\lambda}{\tau_1} \exp\left(-\frac{\lambda}{\tau_1}\right)\right] + \beta_3 \left[\frac{\lambda}{\tau_2} \exp\left(-\frac{\lambda}{\tau_2}\right)\right] \tag{2}$$

$\ddot{\theta} = (\boldsymbol{\beta}, \boldsymbol{\tau})'$, $\boldsymbol{\beta} = (\beta_0,\ \beta_1,\ \beta_2,\ \beta_3)'$, and $\boldsymbol{\tau} = (\tau_1, \tau_2)'$. β_0 is constant value of the yield function, it is always constant if the term maturity approaches zero, β_1 determine the starting values curve (short-term) in various forms of irregularities, curve will be negatively skewed if the parameter is positive and vice versa, β_2 determine the magnitude and direction of the hump, if β_2 positive then hump will occur in τ_1, if β_2 negative then will form a U curve on τ_1, and β_3 together with β_2 which determines the magnitude and direction of the second hump, τ_1 special position of the first hump or U shape curve, τ_2 the second hump position or shape of the curve U. Rezende and Ferreira (23)) added a second slope back into the SV models to obtain the five-factor model (RF model) as follows:

$$y\left(\lambda, \ddot{\theta}\right) = \beta_0 + \beta_1\, exp\left(-\frac{\lambda}{\tau_1}\right) + \beta_2\, exp\left(-\frac{\lambda}{\tau_2}\right) + \beta_3 \left[\frac{\lambda}{\tau_1} \exp\left(-\frac{\lambda}{\tau_1}\right)\right] + \beta_4 \left[\frac{\lambda}{\tau_2} \exp\left(-\frac{\lambda}{\tau_2}\right)\right] \tag{3}$$

$\ddot{\boldsymbol{\beta}} = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)'$, $\boldsymbol{\tau} = (\tau_1, \tau_2)'$, $\ddot{\theta} = \left(\ddot{\boldsymbol{\beta}}, \boldsymbol{\tau}\right)$, parameter bounds are $\beta_0 > 0$, $\beta_0 + \beta_1 > 0$, and $\tau_1, \tau_2 > 0$. SVE model is a model that we have developed from previous models. The development of this model aims to increase the accuracy of the model. The expansion is done by adding a second hump in the third factor of the SV model. This addition is an extension of SV model. This addition is an extension SV model. Expansion of the model is formulated as follows:

$$y(\lambda, \boldsymbol{\theta}) = \beta_0 + \beta_1\, exp\left(-\frac{\lambda}{\tau_1}\right) + \beta_2 \left(\left[\frac{\lambda}{\tau_1} \exp\left(-\frac{\lambda}{\tau_1}\right)\right] + \left[\frac{\lambda}{\tau_2} \exp\left(-\frac{\lambda}{\tau_2}\right)\right]\right) + \beta_3 \left[\frac{\lambda}{\tau_2} \exp\left(-\frac{\lambda}{\tau_2}\right)\right], \tag{4}$$

$\ddot{\boldsymbol{\beta}} = (\beta_0, \beta_1, \beta_2, \beta_3)'$, $\tau = (\tau_1, \tau_2)'$, and $\ddot{\theta} = (\boldsymbol{\beta}, \boldsymbol{\tau})'$. β_0 is constant value of the yield function, it is always constant if the term maturity approaches zero, β_1 determine the starting values curve (short-term) in various forms of irregularities, curve will be negatively skewed if the parameter is positive and vice versa, β_2 determine the magnitude and direction of the hump, This curve will be positively skewed if β_2 positive and negatively skewed if β_2 negative, β_3 determine the shape of the second hump, this curve will be positively skewed if β_3 positive and will be *S*-shaped if β_3, τ_1 determine the position of the first hump and τ_2 determine the position of the second hump.

Data and Estimation

Determining the value of the parameters of each model, we estimate using the R programming by changing the existing functionality in the package L-BFGS-B is accompanied by restrictions on the value of each parameter and determining the initial value of each model parameter first. The selection of these values applies to any data used. In this paper, we use daily data obtained from the Indonesian government bonds. By running the program we obtain the yield curve for each model, namely:

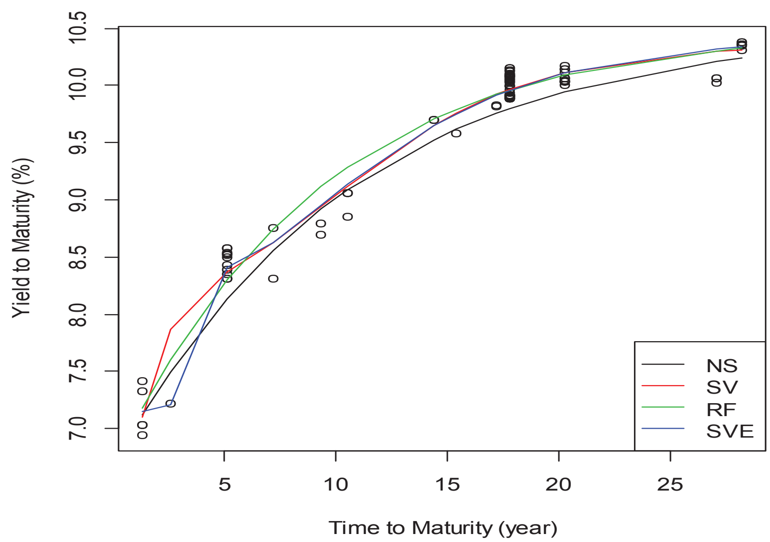

FIGURE 1. Yield curve of NS Class Model on May 5, 2010

These curves show the yield curve of NS class model. NS class model curve shows the yield to maturity by curve depicted from the actual data spread, the difference curve models is evident with each other. This difference shows that the curve is more accurate in describing the yield curve of bond. To determine the best model of these models, we compare the value of the Mean Square Error (MSE), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), Akaike's Information Criterion (AIC), and Bayesian Information Criterion (BIC) as table 1:

Table 1. Evaluation Value the NS Class Model Observation on May 5, 2010

Model	MSE	AIC	BIC
NS	0.023262	-2.67762	-3.57077
SV	0.018698	-2.85435	-3.69408
RF	0.023262	-2.61512	-3.42813
SVE	0.014301	-3.12242	-3.96215

In Table 1 shows that the difference of MSE for SVE models to models of NS, RF, and SVE, respectively are 0.90%, 0.44%, and 0.90%. AIC also showed differences respectively are 44.48%, 26.81%, and 50.73%. BIC has the distinction as follows: 39.14%, 26.81%, and 53.40%. Indicator value differences are concluded that the observations on May 5, 2010 SVE models are the best models in determining the yield curve. To strengthen this conclusion, we observe that occurred on May 14, 2010 as shown in Figure 2.

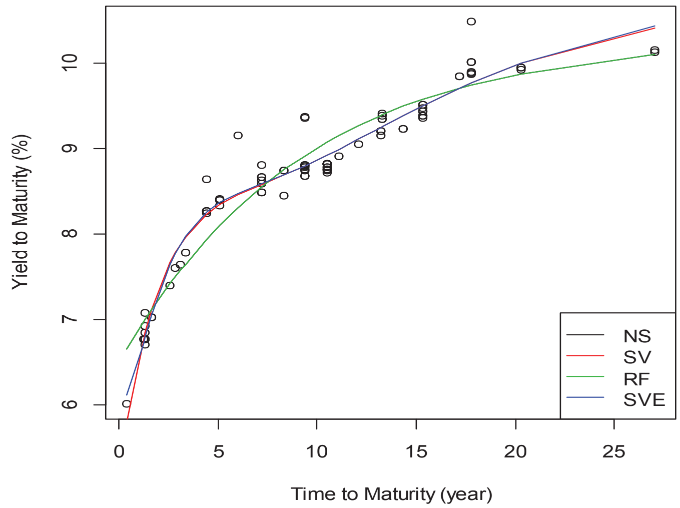

FIGURE 2. Yield Curve NS Class Model on May 14, 2010

In this picture shows curve of NS model coincides with the curve of RF models, the models curve have same curve in determining the yield curve. Different things are shown by curves SV and SVE models. In the second curve shown has a slight difference. The differences are visible at the base and tip of the curve. This difference can also we look at the resulting indicator values in Table 2:

TABLE 2. Evaluation Value the NS Class Model at Observation May 14, 2010.

Model	MSE	AIC	BIC
NS	0.065757	-1.63668	-2.52846
SV	0.034465	-2.24015	-3.07781
RF	0.065757	-1.57285	-2.38346
SVE	0.033813	-2.25925	-3.09692

In this table shows MSE of the model NS and RF models have the same value so that both models can be inferred extension of the NS model become RF models do not provide a more meaningful effect only difference this indicator only looks at the value AIC and BIC, while the SVE models differ for each indicator. This difference can we conclude that the model SVE is the best model. Furthermore, we conduct observations on May 21, 2010; the yield curve is shown in Figure 3:

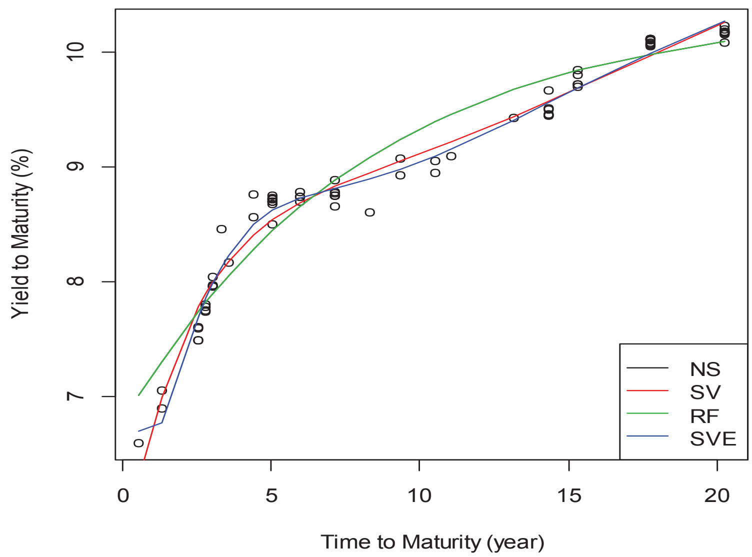

FIGURE 3. Yield Curve of NS Class Model on May 21, 2010

This curve illustrates that the NS and RF models coincide shown in one curve, For SV and SVE models visible difference, These differences confirm that the model curve can be used as a basis in determining the yield curve. To determine the best model, can be considered Table 3:

TABLE 3. Evaluation Value of NS Class Model Group on Observation May 21, 2010.

Model	MSE	AIC	BIC
NS	0.044586	-2.00778	-2.88692
SV	0.020211	-2.7477	-3.56641
RF	0.044586	-1.93086	-2.71936
SVE	0.012661	-3.21538	-4.03409

In this table visible that the NS and RF models has the same value shown in the MSE, AIC, and BIC. SV and SVE models show a more significant difference. The difference value can be shown to the conclusion that the model SVE is the best model on observations of May 21, 2010.

CONCLUSION

The yield curve is a curve illustrating the magnitude of the yield to maturity. This curve modeling can use a variety of models, one of which is the Nelson-Siegel class model. Nelson-Siegel class model discussed were: Nelson-Siegel model, Svensson model, Rezende-Ferreira model and Svensson extended models. Svensson extended model is a model that is extended by combining the first hump and the second hump in one factor. This incorporation aims to find a model that is more accurate in determining the yield curve.

To determine the best model, we observed value indicators such as MSE, 7AIC and BIC of each model. Based on the observation that has been made, show that the observed indicator values of each model, SVE models have an indicator value that is smaller than the model NS, SV, and RF models. This difference is concluded that the SVE models best models in determining the yield curve.

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