ESTIMATING DISCOUNT RATE WITH EXTENDED NELSEN SIEGEL VENSSON MODELS

by Muslim Dedi Rosadi

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***Muslim*[1], *Dedi Rosadi*[2], *Gunardi*[3], *Abdurrahman*[4]**

Abstract Nelson Siegel Svenson model usually is used for yield curve analysis. This model is very complicated because consist of about eight variables, so it will be problem to solve and optimize this model. This paper aims to discuss about solution of Nelson Siegel Svenson model by using Gauss-Newton method and Levenberg-Marquardt algorithms. The solution will be derived theoritically and it will be ilustrated by numerical example.

Keyword: Gauss-Newton, Nonlinear Equation

1. INTRODUCTION

Bond is once instruments in market price, such as bond make the choice investor in invest his wealt in order increase wealth that don't reduce his wealt value. the bond beter is the bond wiches has default risk very small, it is the choice for investors, such as this bond has interest to get its. So, we interest in research develop this bond.

In Rahardjo (2003) bond from issuer consists of three i.e Government Bond, Municipal Bond, and Corporate Bond. Government Bond is bond issued by the central government with the aims to government purposes. Given colateral is the allocation of government revenue derived from taxes or other government revenue. Municipal bond is bond issued by local government for develop public facilities projects in the region. The fund from the bond using to requirement fasilities public. Corporate bond is bond issued by private companies with aims for get fund in develop companies.

In this paper, we were research government bond, because it is one of the factors supporting the development of the State and has a relatively small risk. For interest investors, Government is given an attractive offer interesting for investor, and collateral is the best beter from other bonds.

A widely citied study by Prastowo (2007) find that There are two main things that drives high interest on Government Bonds, i.e because included in group risk-free investment fortfolio and its coupon higher intereste rate Bank. In May 2006 price of government bonds showed a very sharp decline, although at next week bond prices increase. This problem cause exchange of Dolar U. S to Rupiah, and performance IHSG under pressure (Bank Indonesia, 2006).

Due to many factors that influence the price of Government bonds, the researchers tried to see the factor of exchange rate U.S. dollar to rupiah and the Jakarta Stock Exchange (JKSE), researchers estimate that is very dominant factor affecting the yield curve of government bonds.

Various papers examine the effects of macro factors affecting the yield curve model of the bond, such as Landschoot (2004) writes that Nelson Siegel model by adding a liquidity factor and the difference between sample coupon bonds with its average, Diebold, Rudebusch, and Aurora (2005) who wrote about the model yield curve with macroeconomic factors. Among the manufacturing capacity utilization, the federal funds rate, and annual price inflation. Furthermore, Diebold, Piazzesi, and Rudebusch (2005) bond yield curve modeling in a way to construct yield curve factor with restrictions that arbitrage, next Diebold, Li, and Yue (2008) forecasting and modeling of yield curves with the time factor and comparing the level of bonds of developed countries like the United States, Japan, Britain and Germany.

2. MODEL

The basic model presented in this paper was developed by Charles Nelson and Andrew Siegel of the University of Washington in 1987. The Svensson model is an extension of this previous methodology. Since the logic underlying the models is identical, the text will focus on the more sophisticated svensson model.

Contionous interest rate concepts are critically important to any understanding of the Nelson-Siegel and Svensson methodologies. Conseqently, these concepts will be briefly introduced prior to the models being described. In general practice, interest rates are compounded at discrete intervals. In order to construct continuous interest rate function (i.e., zero coupon or forward rate interest rate curve), the coumpounding frequency must also be made continous. It should be noted, however, that the impact on zero coupon and forward rate due to the change from semmi-annual to continuous compounding is not dramatic.

On a continuously compounded basis, the zero-coupon rate $z(t,T)$ can be expressed as a function of discretely compounded zero-coupon rate $Z(t,T)$ and the term to maturity, T, as follows:

$$z(t,T) = \exp\left(\frac{Z(t,T)}{100}.(T-t)/365\right) \quad (1)$$

The countinuosly compounded discount factor can be similarly expressed:

$$disc(t,T) = \exp -\left(\frac{Z(t,T)}{100}(T-t)/365\right) \quad (2)$$

The forward rate can also be refresented as a continuously compounded rate:

$$f(t,\tau,T) = \frac{[(T-t)\cdot z(t,T]-[(\tau-t)\cdot z(t,\tau)]}{T-\tau} \quad (3)$$

Another important concepts is instaneous forward rate $(f(t,\tau,T)_{INST})$. This equation is limit from before equation (see equation 3) as term to maturity with forward rate toward zero:

$$(f(t,\tau,T)_{INST}) = \lim_{t\to T} f(t,\tau,T), \quad (4)$$

Instaneous forward rate can be defined as marginal cost of borrowing (or marginal revenue from lending) for infinitely short period of time. In practice, it would be equivalent to a forward overnight interest rate. Thus, if equation (1) in defferential with respect to time, the following expresssion will be obtained:

$$(f(t,\tau,T)_{INST}) = z(t,T) + (T-t)\cdot\frac{\partial z(t,T)}{dt}, \quad (5)$$

Equivalently, the zero-coupon rate is the integral of the instaneous forward rate in the interval from settlement (time T), devided by the number of periods to determine a period zero-coupon rate. It is summarized as follows:

$$z(t,T) = \frac{\int_{x=t}^{T} f(t,\tau,T)_{INST}\,dx}{T-t}, \quad (6)$$

The important relationship between zero-coupon and instaneous forward rates is a critical component of the Nelson-Siegel and Svensson model.

The Nelson-Siegel and Svensson model is a parametric models wich spesicially function of instaneous forward rate $(f(m))$, it is a time to maturity (m) function as follows:

$$f(m)_t = \beta_{0,t} + \beta_{1,t} \exp - \left(\frac{m_t}{\tau_{1,t}}\right) + \beta_{2,t} \frac{m_t}{\tau_{1,t}} \exp - \left(\frac{m_t}{\tau_{1,t}}\right) + \beta_{3t} \frac{m_t}{\tau_{2,t}} \exp - \left(\frac{m_t}{\tau_{2,t}}\right) + \varepsilon_t, \quad (7)$$

With subsitusion equation (7) to equation (2) is obtained interest rates discount factor function as follows:

$$disc_t(m) = exp - \left[\beta_{0,t} m_t + \beta_{1,t} m_t \left(\frac{1 - exp(-m_t/\tau_{1,t})}{-m_t/\tau_{1,t}}\right) + \beta_{2,t} m_t \left(\frac{1 - exp(-m_t/\tau_{1,t})}{-m_t/\tau_{1,t}} - exp(-m_t/\tau_{1,t})\right) + \beta_{3,t} m_t \left(\frac{1 - exp(-m_t/\tau_{2,t})}{-m_t/\tau_{2,t}} - exp(-m_t/\tau_{2,t})\right) + \varepsilon_{t,} \quad (8)$$

With $\tilde{\varepsilon} \sim N(0, \sigma^2)$

Where $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ is parameter the Nelson-Siegel and Svensson model, for simplify in estimation of parameter from interest rates discount factors function then to be transformation log, such as follows:

$$D_t(m, \boldsymbol{\theta}) = \frac{log(disc_t(m))}{m_t} = \beta_{0,t} + \beta_{1,t} \left(\frac{1 - exp(-m_t/\tau_{1,t})}{-m_t/\tau_{1,t}}\right) + \beta_{2,t} \left(\frac{1 - exp(-m_t/\tau_{1,t})}{-m_t/\tau_{1,t}} - exp(-m_t/\tau_{1,t})\right) + \beta_{3,t} \left(\frac{1 - exp(-m_t/\tau_{2,t})}{-m_t/\tau_{2,t}} - exp(-m_t/\tau_{2,t})\right) + \varepsilon_t \quad (9)$$

with $\varepsilon \sim N(0, \sigma^2)$.

D dan m are $N_t \times 1$ matrices of spot rates and years to maturity, respectly, with N_t the number of bonds at time *t*. $\boldsymbol{\theta}_t = (\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t}, \tau_{2,t})$ is the parameter vector. β_0 refresents the long-run level of interest rates, β_1 the short-run component, and β_2 the medium term component. If the time to maturity goes to infinity, the spot rate converges to β_0. the time to maturity goes to zero, the spot rate converges to $\beta_0 + \beta_1$ should be positive. β_0 can be interpreted as the long-run interest rate and $\beta_0 + \beta_1$ as the instaneous interest rate. This implies that $-\beta_1$ can be positive and vicee versa. β_1 also indicated the speed with which the curve volves towards its long-run trend. β_2 determines the magnitude and the direction of the hump or through in the yield curve. The parameter τ_1 is a time constant that should be positive in order to assure convergence to the long-term value β_0. This parameter specifies the position of the hump or through on the yield curve. The specification in equatian (9) is estimated on a weekly basis on a cross-section of N_t bonds at time *t*. In Lanschoot (2004), find that Nelson-Siegel model adding liquidity factor, difference between the coupon of a bond and the average coupon rate of the sample, dummy for the plus subcategory , and dummy from minus category, based on this problem, we looked affect from foreign exchang US dolar to IDR and Jakarta Stock Exchange (JKSE/IHSG), it is focus in this paper, as follows;

$$D_t(m, \tilde{\boldsymbol{\theta}}) = \beta_{0,t} + \beta_{1,t} \left(\frac{1 - exp(-m_t/\tau_{1,t})}{-m_t/\tau_{1,t}}\right) + \beta_{2,t} \left(\frac{1 - exp(-m_t/\tau_{1,t})}{-m_t/\tau_{1,t}} - exp(-m_t/\tau_{1,t})\right)$$

$$+\beta_{3,t}\left(\frac{1-exp(-m_t/\tau_{2,t})}{-m_t/\tau_{2,t}}-exp(-m_t/\tau_{2,t})\right)$$
$$+\beta_{4,t}KURS_t+\beta_{5,t}IHSG_t+\tilde{\varepsilon}_t \quad (9)$$

With $\tilde{\varepsilon}\sim N(0,\sigma^2)$.

$\beta_0,\beta_1,\beta_2,\beta_3,\tau_1,\tau_2$ represent the parameter in the original NSS model, whereas β_4 and β_5 represent sensitivities of the discount factor to additional factors. The next, we estimate the parameters using nonlinear least square (NLS).

3. ESTIMATION

Based on its before section, we investigated estimator parameter using nonlinear least square (NLS), every set of parameter translates in different discount factor interest rates and bond prices. Therefore, we estimate the parameters as such as to minimize the sum of squared errors between the estimated yields, y^{NSS}, and observed yields to maturity, y, at time t.

$$\widehat{\boldsymbol{\theta}}_t=\arg\min_{\boldsymbol{\theta}_t}\sum_{D=1}^{N_t}(y_t^{NSS}-y_t)^2$$

With N_t the number of bonds at time t. For estimate $\widehat{\boldsymbol{\theta}}$ parameter using nonlinear least square (NLS) with konstrain on its before this paper.

$$\min_{\boldsymbol{\theta}}\sum_{t=1}^{N_t}\left[y_t-\beta_0-\beta_1\left(\frac{1-exp(-m/\tau_1)}{-m/\tau_1}\right)\right.$$
$$-\beta_2\left(\frac{1-exp(-m/\tau_1)}{m/\tau_1}-exp(-m/\tau_1)\right)-\beta_3\left(\frac{1-exp(-m/\tau_2)}{m/\tau_2}\right.$$
$$\left.\left.-exp(m/\tau_2)\right)-\beta_4 kurs+\beta_5 IHSG+\tilde{\varepsilon}\right]^2$$

4. CONCLUSION
5.

In estimation of $\widehat{\boldsymbol{\theta}}$ parameter is dificult problem, such as need optimization of discount factor interest rate function, this context not yet obtained estimator from its function, we hope can solving these problems.

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