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DEVELOPMENT OF STUDENT COMPRHENSION IN CONSTRUCTING LINE AND ROW: ANALYZED FROM APOS THEORY

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ABSTRACT

The way of constructing mathematical concept done by students can be through different ways. The development of student comprehension in constructing that concept can be analyzed by APOS (Action, Processes, Object, and Scheme) Theory. This theory can be used to describe individual's scheme development for certain mathematic topic. This paper discussed how to reveal student ability in constructing or reconstructing action, process, and object of mathematic and organizing it in scheme which is used to solve the problem of line and row.

Keywords: Comprehension, action, process, object and scheme

1. INTRODUCTION

In mathematics, the basic object which is learned is abstract, also often called mental object. Because the object which is abstract, this become one of cause the difficulty to learn mathematic. The underlying aspect in learning mathematic is to implant mathematical concept based on comprehension. Someone who learn mathematic should achieve depth comprehension in order to apply it in real situation and experience the benefit of mathematic in daily life [1].

Comprehension toward a mathematical concept is the result of construction and reconstruction toward mathematical objects [1]. Construction and reconstruction is done through activities of actions, processes, and objects of mathematic which is organized in a scheme to solve mathematical problem. This can be analyzed through analyses of genetic decomposition as operationalization from APOS (Action, Process, Object and Scheme) theory. APOS theory is a constructivist theory about how the achievement of concept learning or mathematical principle occurred, which can be used as elaboration about mental construction from action, process, object and scheme.

This theory can be used as analyses tool to describe one's scheme development toward a mathematical topic which is totality from knowledge related with that object. The scheme development is a process which is dynamic and always changes.

Line and row are one of mathematical topic which is learned in class 1 of Senior High School.

Line and row systems are used much more in our daily life. Line and row concepts also can be used in learning advance mathematic material, for instance algebra function limit. Line concept also can be used to help in finding the pattern, forming the hypothesis, developing critical thinking ability, and prove a mathematical conjecture. Therefore, line and row is one of essential topic, so the correct comprehension is needed [2].

From some researches which had been conducted so far, it can be known that there are still many students who experience the difficulty in comprehending line and row. Therefore it is needed to study how far and what is student doing when he/she construct mathematical concept or solve mathematical problem related with line and row. In this paper, the author discussed the way of tracking student's comprehension of line and row topics. Sub topic discussed is arithmetic line and row so what it means by line and row in this paper is arithmetic line and row.

2. RESULTS AND DISCUSSION

A. Comprehension

Comprehension can be defined as depth understanding. Comprehension is a process to comprehend certain meaning and the ability to use it in another situation [2]. Comprehension about mathematical concept is the result of construction and reconstruction from mathematical objects which is done through activities of action, process, and object which is coordinated in a scheme[1].

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Process of forming scheme involved two activities, namely assimilation and accommodation [3]. Assimilation is process to absorb new experience (knowledge) into existing scheme. Whereas accommodation is process to absorb new experience (knowledge) by modifying existing scheme or even forming experience (knowledge) which is really new. Comprehension in this article defined as student's ability to construct or reconstruct mathematical action, process, and object and organize it in scheme which used to solve the problem of line and row.

B. Mental Constructions in Comprehending Mathematical Concept

Developed APOS theory which is elaboration result of reflective abstraction and which was introduced by Piaget in explaining the development of logical thinking in children. Eexpanded this idea to explain the development of higher order mathematical thinking in college students [1] APOS theory can be used to explain the development of mathematical thinking of each individual, so the author of this paper choose to study and apply APOS theory to explain the development of Senior High School students' mathematical thinking.

APOS theory assumed that mathematical knowledge possessed is the result of interaction outcome with other people and the result of his/her mental constructions in comprehending mathematical ideas. Those mental constructions are: action, process, object and scheme. These are abbreviated as APOS. Frequently, some constructions are reconstruction from existing one, but its reconstruction is not exactly the same with the existing before. The terms construction and reconstruction which is intended here resemble with Piaget's terms namely accommodation and assimilation [4]. APOS theory is very good to be used to understand Students College learning in various mathematic topics in higher education, such as calculus, abstract algebra, statistic, discrete mathematic [1].

Comprehending the mathematical concept was begin by manipulating existing mental construction or manipulating physical object to form action [1]. Action then is interiorized (contemplated) to form the process which then was encapsulated (crystallized) to form the object. The object can be decomposed again to become process. Finally, action, process, and object can be organized in scheme. The following is brief description for each APOS mental construction.

Action is transformation which is experienced to occur in individual thinking as result of stimuli from outside. The stimuli for instance in the form of implementing instructional stages for an operation. This action is activity in the form of physical repetition or mental manipulation which basing algorithm explicitly. Action can be intended as physical or mental transformation from object to obtain another object.

When individual reflect on action which is done repeatedly, then this action was made to become process which is called internal construction. It was made by doing same action, but not directed to external stimulus. Individual who had constructed "process of concept" can decompose or even reverse steps from transformation without really do it. The object was constructed from process when individual reflect on operation, which is applied on process for certain concept, and it's become aware toward process as totality and really can construct that transformation, so that individual encapsulate process to become object.

The collection of action, process, object and another scheme which is connected integrally and organized structurally in individual thinking is called scheme. Constructions that connect action, process, and object separately that produce a scheme is called thematization. This scheme can be relied on to face mathematical problem. The difference between schemes with other mental constructions resembles the difference in biology between organ and cell. Both are object, but organ (scheme) gives necessities in order that cell to be functioned normally. Scheme of individual is whole knowledge which he connected consciously or unconsciously with certain mathematical concept. Individual can posses the scheme for function, the scheme for derivative, etc. The scheme itself can be treated as object and contain in scheme organization in higher level.

C. Scheme Development

APOS theory can be used directly to analyze a researcher data. Researchers can compare subject's success and failure for mathematical task through certain mental construction which they can or cannot do. In searching and analyzing how student learn mathematical concepts, mental construction elements of action, process and object is very essential element to be noticed by researcher. Description that is result from concept analysis in that construct is called genetic decomposition from concept [1]. Whereas some researchers [4] stated that analysis of genetic decomposition is analyses toward structured collection from mental activities of action, process and object which is done by someone to describe how mathematical concept and principle can be developed in one's thinking. Therefore, genetic decomposition is analyses of genetic decomposition in responding a mathematical problem based on APOS theory framework.

Developed APOS theory was used by adaptting Piaget opinion about one's knowledge development [1, 5, 6]. Piaget and Garcia stated that knowledge grow and develop based on certain mechanism which comprise three levels (stages) called *triad.* Based on above explanation, genetic decomposition analyses in this paper and it is defined as analyses toward student comprehension in responding line and row problem by based on APOS theory. Furthermore, triad stages from Piaget and Garcia was used in analyzing student's comprehension level about mathematical concept.

Triad occur in fixed order which is hierarchies and functional. That order is *intra* stage, *inter* stage, and *trans* stage. In intra stage, student concentrate attention on action or operation which can be repeated, but less able to connect action with condition system which make him/her able to broaden the application? The explanations in this stage are still specific. In this stage, student recognizes the object not as important thing, and its form resemble the simple generalization form.

In inter stage, student realize about the relation which is occurred in object and be able to conclude based on initial process with some comprehension, and another operation as a result, or only able to coordinate with the same operations.

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Table 1: Criteria of Student Comprehension about Arithmetic Line

Topic	APOS Theory of Framework	Criteria
Arithmetic Line	**Action**	**Student capable to** - Distinguish a line from another line by noticing pattern from some suku toward a line. -Assert the difference between a line with another line. **Student capable to** -Explain the way to determine certain suku from a line.
	Process	**Student capable to** -Assert the definition of arithmetic line. -Assert example of an arithmetic line.
	Object	-Assert the relation between one suku with another suku arithmetic line. **Student capable to** -Connect action, process, object of another arithmetic line that is linear function graphic.
	Scheme	- Determine certain suku from arithmetic line by connecting action, process, object of arithmetic line.

Table 2: Criteria of Student Comprehension about Arithmetic Row

Topic	APOS Theory of Framework	Criteria
Arithmetic Row	**Action**	**Student capable to** -Distinguish a line with row by noticing number structure. -Assert the difference between line and row.
	Process	**Student capable to** -Explain the way to determine the number of first suku from a line.
	Object	**Student capable to** -Assert definition of arithmetic row. - Assert example of arithmetic row. -Assert example of row which is not arithmetic row.
	Scheme	**Student capable to** -Connect action, process, object of arithmetic row with another mathematical object, namely arithmetic line concept and sigma notation. -Determine the number of certain suku from a line by connecting action, process, object of arithmetic line and linear equation system.

APOS theory of framework for line and row and its criteria has been determined and enlisted in tables 1 & 2. This process make students are able to group a system by using method which incorporate new transformation. In trans stage, student is able to arrange an awareness of scheme completeness and able to achieve new global properties which cannot be accessed in another stage. Student in this stage has ability to construct whole structure which is found (action, process, object, and another scheme) interrelated and form a coherent scheme.

One's scheme development toward a mathematical topic can be described by using APOS theory. Then, scheme development is analyzed to find out triad stages of one's scheme development. According to some researchers [7], student's scheme development in intra stage is indicated by ability to interiorizing an action toward process. Scheme development in inter stage is indicated by ability to encapsulate a process to object.

Whereas student's scheme development in trans stage is indicated by ability to thematizing object to scheme.

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D. Application of APOS Theory Framework to Line and Row

In this section, brief description about application of APOS theory of framework was discussed by analyses of genetic decomposition, which is integrated with triad stages from Piaget and Garcia to line and row concept. The criteria of student's comprehension about line and row based in APOS theory of framework [2] is tabulated in table 1 & 2.

Action:

Action is physical and mental manipulation which can be repeated in transforming object by one way. For example, students is given the problem "How is the sum of 7 first sukus from line 5, 9, 13, 17, 21, ….?" Student action toward that problem which can be done is as follow. Summing numbers in that line until the 7th suku, that is summing 5+9+13+17+21+25+29 = 119. So, student do the activity actively to find the sum of certain suku from a line by way of summing numbers in certain line.

Interiorization: from action to process

Interiorization is a change from a procedural activity to be able to do this activity again in imagining some understanding which is influential toward condition which is resulted. That change is used to distinguish an action from process, that is activity determine a line suku is interiorized as a process where that action will be done, but not really done. For instance, "How is the sum of 7 first suku from line of 5, 9, 13, 17, 21….?"

In interiorizing the seeking of sum of that 7 first suku, student do not do the action, but do it in imagination and capable to explain the process of determining the sum of 7 first suku from that line, even though she/he still use the way of summing numbers which are existed in that line. Therefore, student can imagine and explain that the sum of 7 first suku from line of 5, 9, 13, 17, 21,….is obtained by adding numbers in line, that is 5,9,13,17,21,25,29 = 119. If student has able to tell and explain about the way to obtain answer from determining the sum of a suku by summing numbers which are existed in line, so the level of student comprehension is in the intra stage.

Encapsulation: from process to object

If a process can be transformed by an action then it had been encapsulated to become object. Encapsulation of process to determine a suku from line is indicated when student capable to show that this line has certain properties and characteristic, a line suku has linked with the next suku in certain category. Based on characteristic of known line, student is able to determine whether that line is included into certain line category.

For example, the question is asked "How is the sum of 7 first suku from line of 5, 9, 13, 17, 21,…?" Student who has been encapsulated line as object is able to explain that this line is arithmetic line, because has characteristic of difference between two sukus in order is constant, which is called difference (d), that is b = 4, and first suku (u_1) = 5, therefore the sum of 7 first suku can be determined by using formula which is obtained from definition of arithmetic row, that is $S_7 = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7$ or $S_7 = 5+9+13+17+21+25+29 = 119$.

If student capable to determine the sum of certain suku from line by noticing the characteristic of line and relate it by concept of certain line and row, and able to give example and non example from row and line, so the level of student comprehension is in the inter stage.

Thematization: from object to scheme

Thematization is a construction and which is connected with action, process, and object and which is separated for certain object in order to generate a scheme. Thematization of first n sum from line as a scheme involved special relation between rows of line with sigma notation. A student is able to thematize the row of line as scheme, if she/he can show the sum of first suku n from a line by linking it with sigma notation. For example, she is given the problem "How is the sum of 7 first suku from line of 5, 6, 13, 17, 21, ….?"

Student who thematize arithmetic line is able to explain that the sum of 7 first suku from that line is a process of seeking the sum of 7 first suku from arithmetic line, because that line pattern has characteristic of arithmetic line, and capable to link the sum of 7 first suku with concept of sigma notation, that is $S_7 = \sum_{i=1}^{7} u_i = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7$

atau $S_7 = 5+9+13+17+21+25+29 = 119$.

Students who capable to link the relation between concept of arithmetic line row with another object (which is sigma notation) have comprehension in trans level.

3. CONCLUSION

Based on APOS theory, student comprehension about line and row problem can be tracked. By genetic decomposition analysis, the development of student comprehension can be revealed. Scheme development of each student for line and row is different, so its scheme development can be mapped into one of triad stage. The study result in revealing the development of student comprehension can be followed up in learning-teaching process activity by choosing appropriate method for learning in class by noticing where the point of student difficulty/inability in comprehending line and row problem.

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